

**Exercise 3**

**Objective function:** Max:  $Z = x_1 + 2x_2$

**subject to:**

$$2x_1 + 4x_2 \leq 20$$

$$x_1 + x_2 \leq 8$$

and  $x_1, x_2 \geq 0$

$$Z - x_1 - 2x_2 = 0$$

$$2x_1 + 4x_2 + S_1 \leq 20$$

$$x_1 + x_2 + S_2 \leq 8$$

and  $x_1, x_2, S_1, S_2 \geq 0$

**Solving with the SIMPLEX:**

		z	x1	x2	s1	s2	RHS	ratio	
	<b>R0</b>	z	1	-1	-2	0	0	0	-
$R1 * 1/4$	<b>R1</b>	s1	0	2	4	1	0	20	5
	<b>R2</b>	s2	0	1	1	0	1	8	8

		z	x1	x2	s1	s2	RHS	ratio	
$R0 - (-2) * R1$	<b>R0</b>	z	1	0	0	0.5	0	10	-
	<b>R1</b>	x2	0	0.5	1	0.25	0	5	10
$R2 - (1) * R1$	<b>R2</b>	s2	0	0.5	0	-0.25	1	3	6

**non-basic variables:** x1 s1      **basic variables:** x2 s2

Having a non-basic variable with a coeff of zero in R0 is indicative of multiple solutions.

Solution A:  $(x_1, x_2) = (0, 5)$  and  $Z = 10$ .

if

we now force x1 to enter the basis we'll see that the solution will lead to the same Z value

		z	x1	x2	s1	s2	RHS	
	<b>R0</b>	z	1	0	0	0.5	0	10
$R1 - (0.5) * R2$	<b>R1</b>	x2	0	0	1	0.5	-1	2
$R2 * 2$	<b>R2</b>	x1	0	1	0	-0.5	2	6

Solution B:  $(x_1, x_2, s_1, s_2) = (2, 6, 0, 0)$  and  $Z = 10$

Solution A:  $(x_1, x_2, s_1, s_2) = (0, 5, 0, 3)$  and  $Z = 10$ .

**Exercise 4**

**Objective function:** Max:  $Z = x_1 + x_2$

**subject to:**

$$x_1 + x_2 \leq 4$$

$$2x_1 + x_2 \leq 6$$

$$x_1 + 2x_2 \leq 6$$

and  $x_1, x_2 \geq 0$

$$Z - x_1 - x_2 = 0$$

$$x_1 + x_2 + S_1 \leq 4$$

$$2x_1 + x_2 + S_2 \leq 6$$

$$x_1 + 2x_2 + S_3 \leq 6$$

and  $x_1, x_2, S_1, S_2, S_3 \geq 0$

**Solving with the SIMPLEX:**

*Tie in entering variable: arbitrary choice - let's choose  $x_1$*

		z	x1	x2	s1	s2	s3	RHS	ratio
R0	z	1	-1	-1	0	0	0	0	-
R1	s1	0	1	1	1	0	0	4	4
R2	s2	0	2	1	0	1	0	6	3
R3	s3	0	1	2	0	0	1	6	6

*R2\*1/2*

		z	x1	x2	s1	s2	s3	RHS
R0	z	1						
R1	s1	0						
R2	X1	0	1	0.5	0	0.5	0	3
R3	s3	0						

*Tie in leaving variable: arbitrary choice - let's choose  $s_1$*

		z	x1	x2	s1	s2	s3	RHS	ratio
R0	z	1	0	-0.5	0	0.5	0	3	-
R1	s1	0	0	0.5	1	-0.5	0	1	2
R2	X1	0	1	0.5	0	0.5	0	3	6
R3	s3	0	0	1.5	0	-0.5	1	3	2

*R1\*2*

		z	x1	x2	s1	s2	s3	RHS
R0	z	1						
R1	X2	0	0	1	2	-1	0	2
R2	X1	0						
R3	s3	0						

*R0-(-0.5)\*R1*  
*R2-(0.5)\*R1*  
*R3-(1.5)\*R1*

		z	x1	x2	s1	s2	s3	RHS
R0	z	1	0	0	1	0	0	4
R1	X2	0	0	1	2	-1	0	2
R2	X1	0	1	0	-1	1	0	2
R3	s3	0	0	0	-3	1	1	0

*Tie in leaving variable leads to a basic variable =0 in the next tableau (S3), thus we're in the presence of a degenerate solution*

**Exercise 5**

**Objective function:** Max:  $Z = x_1 + x_2$

**subject to:**

$$x_1 \leq 10$$

$$x_1 - 3x_2 \leq 15$$

$$x_1 - x_2 \leq 20$$

and  $x_1, x_2 \geq 0$

$$Z - x_1 - x_2 = 0$$

$$x_1 + S_1 \leq 10$$

$$x_1 - 3x_2 + S_2 \leq 15$$

$$x_1 - x_2 + S_3 \leq 20$$

and  $x_1, x_2, S_1, S_2, S_3 \geq 0$

**Solving with the SIMPLEX:**

*Tie in entering variable: arbitrary choice - let's choose  $x_1$*

		z	x1	x2	s1	s2	s3	RHS	ratio
<b>R0</b>	z	1	-1	-1	0	0	0	0	-
<b>R1</b>	x1	0	1	0	1	0	0	10	10
<b>R2</b>	s2	0	1	-3	0	1	0	15	15
<b>R3</b>	s3	0	1	-2	0	0	1	20	20

		z	x1	x2	s1	s2	s3	RHS	ratio
<i>R0-(-1)*R1</i>	<b>R0</b>	z	1	0	-1	1	0	10	-
	<b>R1</b>	x1	0	1	0	1	0	10	-
<i>R2-(1)*R1</i>	<b>R2</b>	s2	0	0	-3	-1	1	5	-1.7
<i>R3-(1)*R1</i>	<b>R3</b>	s3	0	0	-2	-1	1	10	-5

The 2nd entering variable is  $x_2$  (please note all coeff. in this column are either negative or zero, a sign we might have problems with the min ratio test. Then we confirm we do not meet the min ratio test criteria: choosing the smallest positive value (all values are either zero or negative). Thus we are in the presence of an unlimited solution and we say we have an unbound problem

## Exercise 6

**Objective function:** Min:  $Z = 2x_1 - 3x_2 - 4x_3$   
**subject to:**

$$x_1 + 5x_2 - 3x_3 \leq 15$$

$$x_1 + x_2 + x_3 \leq 11$$

$$5x_1 - 6x_2 + x_3 \leq 4$$

and  $x_1, x_2, x_3 \geq 0$

$$Z - 2x_1 + 3x_2 + 4x_3 = 0$$

$$x_1 + 5x_2 - 3x_3 + S_1 \leq 15$$

$$x_1 + x_2 + x_3 + S_2 \leq 11$$

$$5x_1 - 6x_2 + x_3 + S_3 \leq 4$$

and  $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Please note we will solve this as minimization problem, thus we'll have to reverse the optimality and entering var. criteria

In forestry problems it is not common to get a negative value for the objective function, but it is not impossible. After the class I got thinking and the problem could be, for example, related to the minimizing the temperature so that a certain bacteria wouldn't be able to replicate.

## Solving with the SIMPLEX:

Because the problem is solved as a minimization problem, we'll choose as entering variable the one with the biggest positive value ( $x_3$ ). If we were maximizing the choice would fall on the one with the smallest negative value. The criteria for the min ratio test remains unchanged!

		z	x1	x2	x3	s1	s2	s3	RHS	ratio	
$R_0 - (-1) * R_1$	<b>R0</b>	z	1	-2	3	4	0	0	0	0	-
	<b>R1</b>	s1	0	1	5	-3	1	0	15	-5	
$R_2 - (1) * R_1$	<b>R2</b>	s2	0	1	1	1	0	1	11	11	
$R_3 - (1) * R_1$	<b>R3</b>	s3	0	5	-6	1	0	0	1	4	

		z	x1	x2	x3	s1	s2	s3	RHS	ratio
$R_0 - (4) * R_3$	<b>R0</b>	z	1	-22	27	0	0	-4	-16	-
$R_1 - (-3) * R_3$	<b>R1</b>	s1	0	16	-13	0	1	3	27	-2
$R_2 - (1) * R_3$	<b>R2</b>	s2	0	-4	7	0	1	-1	7	1
	<b>R3</b>	x3	0	5	-6	1	0	1	4	-1

The next entering variable is  $x_2$  (biggest positive value) and the leaving var is  $S_2$

		z	x1	x2	x3	s1	s2	s3	RHS	
	<b>R0</b>	z	1							
	<b>R1</b>	s1	0							
$R_2 * 1/7$	<b>R2</b>	x2	0	-0.6	1	0	0	0.1	-0.1	1
	<b>R3</b>	x3	0							

		z	x1	x2	x3	s1	s2	s3	RHS
$R_0 - (27) * R_2$	<b>R0</b>	z	1	-6.6	0	0	-3.9	-0.1	-43
$R_1 - (-13) * R_2$	<b>R1</b>	s1	0	8.6	0	1	1.9	1.1	40
	<b>R2</b>	x2	0	-0.6	1	0	0.1	-0.1	1
$R_3 - (-6) * R_2$	<b>R3</b>	x3	0	1.6	0	1	0.9	0.1	10

There are no more positive coeff in  $R_0$ , thus we've reached the optimal solution is  $(x_1, x_2, x_3, S_1, S_2, S_3) = (0, 1, 10, 40, 0, 0)$  and  $Z = -43$ . All resources were used for  $s_2$  and  $s_3$  but 40 resource units were left unused for the resource represented by constraint 1 ( $S_1 = 40$ )

### Exercise 7

**Objective function:** Max:  $Z = 10x_1 + 30x_2$

**subject to:**

$$x_1 \leq 15$$

$$x_1 - x_2 \leq 20$$

$$-3x_1 + x_2 \geq -30$$

and  $x_1 \geq 0 \quad x_2 \leq 0$

solve the negative RHS:

$$3x_1 - x_2 \leq 30$$

solve the nonpositive  $x_2$ :

$$x_2' = -x_2 \quad \text{with} \quad x_2' \geq 0$$

the new problem will be:

**Objective function:** Max:  $Z = 10x_1 - 30x_2'$

**subject to:**

$$x_1 \leq 15$$

$$x_1 + x_2' \leq 20$$

$$3x_1 + x_2' \leq 30$$

and  $x_1 \quad x_2' \geq 0$

the standard problem for the new model:

$$Z - 10x_1 + 30x_2'$$

$$x_1 + s_1 \leq 15$$

$$x_1 + x_2' + s_2 \leq 20$$

$$3x_1 + x_2' + s_3 \leq 30$$

and  $x_1 \quad x_2' \quad s_1 \quad s_2 \quad s_3 \geq 0$

### Solving with the SIMPLEX:

			z	x1	x2'	s1	s2	s3	RHS	ratio
$R0 - (-1) * R1$	<b>R0</b>	z	1	-10	30	0	0	0	0	-
	<b>R1</b>	s1	0	1	0	1	0	0	15	15
$R2 - (1) * R1$	<b>R2</b>	s2	0	1	1	0	1	0	20	20
$R3 - (1) * R1$	<b>R3</b>	s3	0	3	1	0	0	1	30	10

			z	x1	x2'	s1	s2	s3	RHS
	<b>R0</b>	z	1						
	<b>R1</b>	s1	0						
	<b>R2</b>	s2	0						
$R3 * 1/3$	<b>R3</b>	x1	0	1	0.3	0	0	0.3	10

			z	x1	x2'	s1	s2	s3	RHS
$R0 - (-10) * R3$	<b>R0</b>	z	1	0	33.3	0	0	3.3	100
$R1 - (1) * R3$	<b>R1</b>	s1	0	0	-0.3	1	0	-0.3	5
$R2 - (1) * R3$	<b>R2</b>	s2	0	0	0.7	0	1	-0.3	10
	<b>R3</b>	x1	0	1	0.3	0	0	0.3	10

There are no more negative coeff in R0, thus we've reached the optimal solution is  $(x_1, x_2', s_1, s_2, s_3) = (10, 0, 5, 10, 0)$  and  $Z = 100$ . Please note that since  $x_2' = -x_2$  thus  $x_2 = 0$ . All resources were used for constraint 3, but 5 and 10 resource units were left unused for the resources represented by constraints 1 and 2 ( $s_1 = 5, s_2 = 10$ )

Solution to the real problem  $(x_1, x_2, s_1, s_2, s_3) = (10, 0, 5, 10, 0), Z = 100$

**Exercise 8**

**Objective function:** Max:  $Z = -x_2$   
**subject to:**  
 $x_1 + x_2 + x_3 \leq 100$   
 $x_1 - 5x_2 \leq 40$   
 $x_3 \geq -10$   
 and  $x_1 \geq 0$   $x_2 \leq 0$   $x_3$  unbounded

solve the negative RHS:

$$-x_3 \leq 10$$

solve the nonpositive  $x_2$ :

$$x_2' = -x_2 \text{ with } x_2' \geq 0$$

solve the unbounded  $x_3$ :

$$x_3 = x_3^+ - x_3^-$$

the standard problem for the new model:

$$Z + x_2 \rightarrow Z - x_2' = 0$$

$$x_1 - x_2' + x_3^+ - x_3^- + s_1 = 100$$

$$x_1 + 5x_2' + s_2 = 40$$

$$-x_3^+ + x_3^- + s_3 = 10$$

$$\text{and } x_1, x_2', x_3^+, x_3^-, s_1, s_2, s_3 \geq 0$$

**Solving with the SIMPLEX:**

The entering variable is  $x_2'$  (negative value) and the leaving var is  $s_2$

		z	x1	x2'	x3+	x3-	s1	s2	s3	RHS	ratio
<b>R0</b>	z	1	0	-1	0	0	0	0	0	0	-
<b>R1</b>	s1	0	1	-1	1	-1	1	0	0	100	-100
<b>R2</b>	s2	0	1	5	0	0	0	1	0	40	8
<b>R3</b>	s3	0	0	0	-1	1	0	0	1	10	-

$R2 * 1/5$

		z	x1	x2'	x3+	x3-	s1	s2	s3	RHS	ratio
<b>R0</b>	z	1									
<b>R1</b>	s1	0									
<b>R2</b>	s2	0	0.2	1	0	0	0	0.2	0	8	8
<b>R3</b>	s3	0									

$R0 - (-1) * R2$   
 $R1 - (-1) * R2$

		z	x1	x2'	x3+	x3-	s1	s2	s3	RHS	ratio
<b>R0</b>	z	1	0.2	0	0	0	0	0.2	0	8	
<b>R1</b>	s1	0	1.2	0	1	-1	1	0.2	0	108	
<b>R2</b>	x2'	0	0.2	1	0	0	0	0.2	0	8	
<b>R3</b>	s3	0	0	0	-1	1	0	0	1	10	

There are no more negative coeff in R0, thus we've reached the optimal solution:

$$(x_1, x_2', x_3^+, x_3^-, s_1, s_2, s_3) = (0, 8, 0, 0, 108, 0, 10) \text{ and } Z = 8$$

Please note that: since  $x_2' = -x_2$  and  $x_3 = x_3^+ - x_3^-$ , then  $x_2 = -8$  and  $x_3 = 0$ ; the solution for the original problem is:

$$(x_1, x_2, x_3, s_1, s_2, s_3) = (0, -8, 0, 108, 0, 10) \text{ and } Z = -x_2, Z = -(-8) \Rightarrow Z = 8$$

Not all resources were used for constraints 1 and 3 (108 and 10 units of each resource were respectively left unused:  $s_1 = 108$  and  $s_3 = 10$ )

Exercise 9

Solving with the SIMPLEX:

Objective function: Min:  $Z = 4x_1 + 2x_2$   
 subject to:

$$2x_1 - x_2 \geq 4$$

$$x_1 + x_2 \geq 5$$

and  $x_1, x_2 \geq 0$

$$Z = 4x_1 + 2x_2 + M a_1 + M a_2$$

$$2x_1 - x_2 - S_1 + a_1 = 4$$

$$x_1 + x_2 - S_2 + a_2 = 5$$

and  $x_1, x_2, S_1, S_2, a_1, a_2 \geq 0$

Where  $a_1 = 4 - 2x_1 + x_2 + S_1$

$$a_2 = 5 - x_1 - x_2 + S_2$$

Replacing in the OF:  $\square$

$$\text{Min } Z = 4x_1 + 2x_2 + M(4 - 2x_1 + x_2 + S_1) + M(5 - x_1 - x_2 + S_2)$$

$$\text{Min } Z = 4x_1 + 2x_2 + 4M - 2Mx_1 + Mx_2 + MS_1 + 5M - Mx_1 - Mx_2 + MS_2$$

$$\text{Min } Z = 4x_1 - 2Mx_1 - Mx_1 + 2x_2 + Mx_2 - Mx_2 + MS_1 + MS_2 + 4M + 5M$$

$$\text{Min } Z = 4x_1 - 3Mx_1 + 2x_2 + MS_1 + MS_2 + 9M$$

$$\text{Min } Z = (4 - 3M)x_1 + 2x_2 + MS_1 + MS_2 + 9M$$

$$Z - (4 - 3M)x_1 - 2x_2 - MS_1 - MS_2 = 9M$$

Because this is a minimization problem the selection criteria for the entering variable is finding the biggest positive value in R0, thus X1 is the entering variable. The criteria for the leaving variable remains unaltered

		z	x1	x2	s1	s2	a1	a2	RHS	ratio
R0	Min z	1	3M - 4	-2	-M	-M	0	0	9M	-
R1	a1	0	2	-1	-1	0	1	0	4	2
R2	a2	0	1	1	0	-1	0	1	5	5

R1\*1/2

		z	x1	x2	s1	s2	a1	a2	RHS
R0	Min z								
R1	x1	0	1	-0.5	-0.5	0	0.5	0	2
R2	a2	0							

R0-(3M-4)\*R1

R1\*1/2

R2-(1)\*R1

		z	x1	x2	s1	s2	a1	a2	RHS
R0	Min z	1	0	-4 + 3/2M	1/2M - 2	-M	-3/2M + 2	0	3M + 8
R1	x1	0	1	-0.5	-0.5	0	0.5	0	2
R2	a2	0	0	1.5	0.5	-1	-0.5	1	3

R0-(3M-4)\*R1:

$$x_1 \quad (3M - 4) - (3M - 4)*1 = 0 \quad 3M - 4 - 3M + 4 = 3M - 3M - 4 + 4 = 0$$

$$x_2 \quad -2 - (3M - 4)*(-0.5) = -2 - (-3/2M + 2) = -4 + 3/2M$$

$$s_1 \quad -M - (3M - 4)*(-0.5) = -M - (-3/2M + 2) = -M + 3/2M - 2 = 1/2M - 2$$

$$s_2 \quad -M - (3M - 4)*(0) = -M$$

$$a_1 \quad 0 - (3M - 4)*(0.5) = -3/2M + 2$$

$$a_2 \quad 0 - (3M - 4)*(0) = 0$$

$$RHS \quad 9M - (3M - 4)*(2) = 9M - 6M + 8 = 3M + 8$$

We haven't reached the optimal solution because there are still 2 positive coeff (x2 and S1), so we

Exercise 9 (cont.)

choose the biggest positive one

Solving with the SIMPLEX:

		z	x1	x2	s1	s2	a1	a2	RHS	ratio
R0	Min z	1	0	-4 + 3/2M	1/2M - 2	-M	-3/2M + 2	0	3M + 8	-
R1	x1	0	1	-0.5	-0.5	0	0.5	0	2	-4
R2	a2	0	0	1.5	0.5	-1	-0.5	1	3	2

R2\*2/3

		z	x1	x2	s1	s2	a1	a2	RHS
R0	Min z	1							
R1	x1	0							
R2	x2	0	0	1	0.3	-0.7	-0.3	0.7	2

R0 - (-4 + 3/2M)\*R2  
R1 - (-0.5)\*R2

		z	x1	x2	s1	s2	a1	a2	RHS
R0	Min z	1	0	0	-0.66667	-8/3	-M + 2/3	8/3 - M	16
R1	x1	0	1	0	-0.3	-0.3	0.3	0.3	3
R2	x2	0	0	1	0.3	-0.7	-0.3	0.7	2

R0 - (3M - 4)\*R1:

x1            0

x2            0

s1             $(1/2M - 2) - (-4 + 3/2M)*1/3 = 1/2M - 2 - (-4/3 + 2/3M*1/3) = (-6 + 4)/2 = -2/3$

s2             $-M - (-4 + 3/2M)*(-2/3) = -M - (4*2)/3 - 3/2*2/3M = -M - 8/3 + M = -8/3$

a1             $-3/2M + 2 - (3/2M - 4)*(-1/3) = -3/2M + 2 - (-3/2*1/3M + 4/3) = -3/2M + 2 + 1/2M - 4/3 = -M + 2/3$

a2             $0 - (3/2M - 4)*(2/3) = -(3/2*2/3M - 4*2/3) = -(M - 8/3) = 8/3 - M$

RHS             $3M + 8 - (3/2M - 4)*2 = 3M + 8 - (3M - 8) = 3M - 3M + 8 + 8 = 16$

There are no more positive coeff in R0, thus we've reached the optimal solution:

$(x1, x2, S1, S2) = (3, 2, 0, 0)$     and     $Z = 16$



**Exercise 11**

**Max:**  $Z = 3x_1 + 2x_2$   
**Subject to:**  $2x_1 + x_2 \leq 9$   
 $x_1 + 2x_2 \leq 9$

$x_1, \geq 0$ ;  $x_2$  unbounded

$x_2 = x_2' - x_2''$   $x_1, x_2', x_2'' \geq 0$

**Max:**  $Z = 3x_1 + 2(x_2' - x_2'')$   
**Subject to:**  $2x_1 + (x_2' - x_2'') \leq 9$   
 $x_1 + 2(x_2' - x_2'') \leq 9$

$x_1, x_2', x_2'' \geq 0$

**Max:**  $Z = 3x_1 + 2x_2' - 2x_2''$   
**Subject to:**  $2x_1 + x_2' - x_2'' \leq 9$   
 $x_1 + 2x_2' - 2x_2'' \leq 9$

$x_1, x_2', x_2'' \geq 0$

**Max:**  $Z - 3x_1 - 2x_2' + 2x_2''$   
**Subject to:**  $2x_1 + x_2' - x_2'' + S_1 = 9$   
 $x_1 + 2x_2' - 2x_2'' + S_2 = 9$

$x_1, x_2', x_2'', S_1, S_2 \geq 0$

entering

basic var	z	x1	x2'	x2''	s1	s2	RHS
z	1	-3	-2	2	0	0	0
s1	0	2	1	-1	1	0	9
s2	0	1	2	-2	0	1	9

$(x_1, x_2', x_2'', s_1, s_2) = (0, 0, 0, 9, 9)$   
 $z = 0$   
 leaving: 4.5  
 9      A:  $(x_1, x_2) = (0, 0)$

entering

basic var	z	x1	x2'	x2''	s1	s2	RHS
z	1	0	-0.5	0.5	1.5	0	13.5
x1	0	1	0.5	-0.5	0.5	0	4.5
s2	0	0	1.5	-1.5	-0.5	1	4.5

$(x_1, x_2', x_2'', s_1, s_2) = (4.5, 0, 0, 0, 4.5)$   
 $z = 13.5$   
 leaving: 9  
 3      B:  $(x_1, x_2) = (4.5, 0)$

basic var	z	x1	x2'	x2''	s1	s2	RHS
z	1	0	0	0	1.3333	0.3333	15
x1	0	1	0	0	0.6667	-0.333	3
x2'	0	0	1	-1	-0.333	0.6667	3

$(x_1, x_2', x_2'', s_1, s_2) = (3, 3, 0, 0, 0)$   
 $z = 15$   
 optimal solution !      C:  $(x_1, x_2) = (3, 3)$

Solution to the real problem

$x_2 = x_2' - x_2''$   
 $x_2 = 3 - 0$

$(x_1, x_2, s_1, s_2) = (3, 3, 0, 0)$

**Exercise 12**

**Max:**  $Z = 3x_1 + 4x_2$   
**Subject to:**  $x_1 + x_2 \leq 10$   
 $5x_1 + 12x_2 \leq 60$   
 $x_1 + 3x_2 \leq 15$   
 $x_2 \leq 5$   
 $x_1, x_2 \geq 0$

basic var	z	x1	x2	s1	s2	s3	s4	RHS	1, x2,s1,s2, s3,s4)
z	1	-3	-4	0	0	0	0	0	z = 0
s1	0	1	1	1	0	0	0	10	10
s2	0	5	12	0	1	0	0	60	5
s3	0	1	3	0	0	1	0	15	5
s4	0	0	1	0	0	0	1	5	5

entering  
leaving  
A: (x1,x2)= (0,0)

**Max:**  $Z = 3x_1 - 4x_2$   
 $x_1 + x_2 + s1 = 10$   
 $5x_1 + 12x_2 + s2 = 60$   
 $x_1 + 3x_2 + s3 = 15$   
 $x_2 + s4 = 5$   
 $x_1, x_2, s1, s2, s3, s4 \geq 0$

basic var	z	x1	x2	s1	s2	s3	s4	RHS	1, x2,s1,s2, s3,s4)
z	1	-1.333	0	0	0.333	0	0	20	z = 20
s1	0	0.583	0	1	-0.083	0	0	5	8.57143
x2	0	0.417	1	0	0.083	0	0	5	12
s3	0	-0.25	0	0	-0.25	1	0	0	-
s4	0	-0.417	0	0	-0.083	0	1	0	-

entering  
leaving  
B: (x1,x2)= (0,5)

basic var	z	x1	x2	s1	s2	s3	s4	RHS	1, x2,s1,s2, s3,s4)
z	1	0	0	2.286	0.143	0	0	31.4	z = 31.428571
x1	0	1	0	1.714	-0.143	0	0	8.6	C: (x1,x2)= (8.6,1.4)
x2	0	0	1	-0.714	0.143	0	0	1.4	
s3	0	0	0	0.429	-0.286	1	0	2.1	
s4	0	0	0	0.714	-0.143	0	1	3.6	optimal solution!!

When facing the tie in the leaving variable for the 1st table if we add chosen a different leaving variable it would have taken us longer (more tables) to solve the problem. I've chosen to take out the slack corresponding to the binding constraint (one of the two constraints that determine the optimal cornerpoint). Of course I only knew this because I looked at the graph. For more complex problems when graphing is not possible we choose arbitrarily.

Exercise 12 (cont.)

choosing a different leaving variable:

		entering								1, x2, s1, s2, s3, s4) =
		z	x1	x2	s1	s2	s3	s4	RHS	(0,0,10,60,15,5)
z		1	-3	-4	0	0	0	0	0	z = 0
s1		0	1	1	1	0	0	0	10	10
s2		0	5	12	0	1	0	0	60	5
s3		0	1	3	0	0	1	0	15	5
leaving s4		0	0	1	0	0	0	1	5	5

A:  
(x1,x2) = (0,0)

look what happens if we force s2 or s3 to leave the basis (despite these have ratios of 0, shouldn't even be candidates to leave):

		entering								1, x2, s1, s2, s3, s4) =
		z	x1	x2	s1	s2	s3	s4	RHS	(0,5,5,0,0,0)
z		1	-3	0	0	0	0	4	20	z = 20
s1		0	1	0	1	0	0	-1	5	5
s2		0	5	0	0	1	0	-12	0	0
leaving s3		0	1	0	0	0	1	-3	0	0
x2		0	0	1	0	0	0	1	5	-

B:  
(x1,x2) = (0,5)  
degenerate solution because we have 2 basic variables = 0!  
Still at point B

		entering								1, x2, s1, s2, s3, s4) =
		z	x1	x2	s1	s2	s3	s4	RHS	(0,5,5,0,0,0)
z		1	0	0	0	0	3	-5	20	z = 20
s1		0	0	0	1	0	-1	2	5	2.5
leaving s2		0	0	0	0	1	-5	3	0	0
x1		0	1	0	0	0	1	-3	0	0
x2		0	0	1	0	0	0	1	5	5

B:  
(x1,x2) = (0,5)  
degenerate solution because we have 2 basic variables = 0!  
Still at point B

		entering								1, x2, s1, s2, s3, s4) =
		z	x1	x2	s1	s2	s3	s4	RHS	(0,5,5,0,0,0)
z		1	0	0	0	1.667	-5.333	0	20	z = 20
leaving s1		0	0	0	1	-0.667	2.333	0	5	2.14286
s4		0	0	0	0	0.333	-1.667	1	0	0
x1		0	1	0	0	1	-4	0	0	0
x2		0	0	1	0	-0.333	1.667	0	5	3

B:  
(x1,x2) = (0,5)  
degenerate solution because we have 2 basic variables = 0!  
Still at point B

Exercise 12 (cont.)

	z	x1	x2	s1	s2	s3	s4	RHS
z	1	0	0	2.286	0.143	0	0	31.4
s3	0	0	0	0.429	-0.286	1	0	2.1
s4	0	0	0	0.714	-0.143	0	1	3.6
x1	0	1	0	1.714	-0.143	0	0	8.6
x2	0	0	1	-0.714	0.143	0	0	1.4

$$1, x_2, s_1, s_2, s_3, s_4) = (8.6, 1.4, 0, 0, 2.1, 3.6)$$

$$z = 31.428571$$

C:

$$(x_1, x_2) = (8.6, 1.4)$$

optimal solution!!



















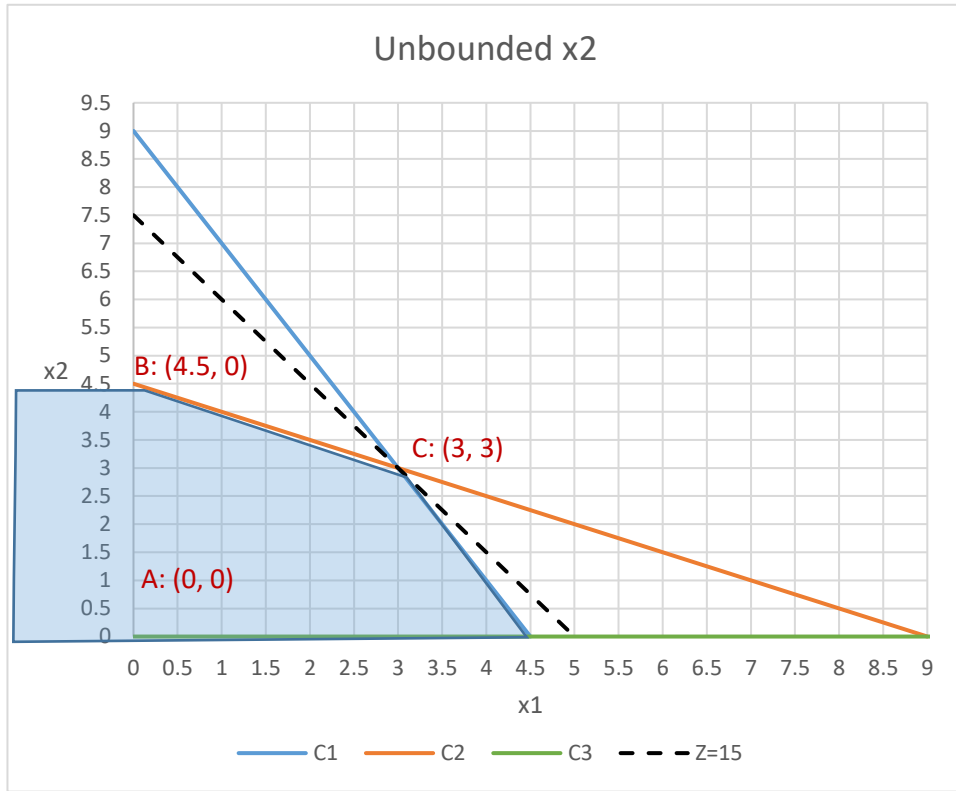
Exercise 11

$$\begin{array}{rcll} \max z = & 3x_1 & + & 2x_2 \\ Z=15 & 15 & & 0 & 7.5 \\ & & & 5 & 0 \end{array}$$

$$\begin{array}{rcll} \text{C1} & 2x_1 & + & 1x_2 & = & 9 \\ & 0 & & 9 & & \\ & 4.5 & & 0 & & \end{array}$$

$$\begin{array}{rcll} \text{C2} & 1x_1 & + & 2x_2 & = & 9 \\ & 0 & & 4.5 & & \\ & 9 & & 0 & & \end{array}$$

$$\begin{array}{rcll} \text{C3} & x_1 & & & = & 0 \\ & 10 & & 0 & & \\ & 0 & & 0 & & \end{array}$$



## Exercise 12

$$\max z = 3x_1 + 4x_2$$

$$Z=31.4 \quad 35 \quad 0 \quad 8.8$$

$$12 \quad 0$$

$$39 \quad 1.4 \quad 8.6$$

$$C1 \quad 1x_1 + 1x_2 = 10$$

$$0 \quad 10$$

$$10 \quad 0$$

$$C2 \quad 5x_1 + 12x_2 = 60$$

$$0 \quad 5$$

$$12 \quad 0$$

$$C3 \quad 1x_1 + 3x_2 = 15$$

$$0 \quad 5$$

$$15 \quad 0$$

$$C4 \quad x_2 = 5$$

$$0 \quad 5$$

$$15 \quad 5$$

